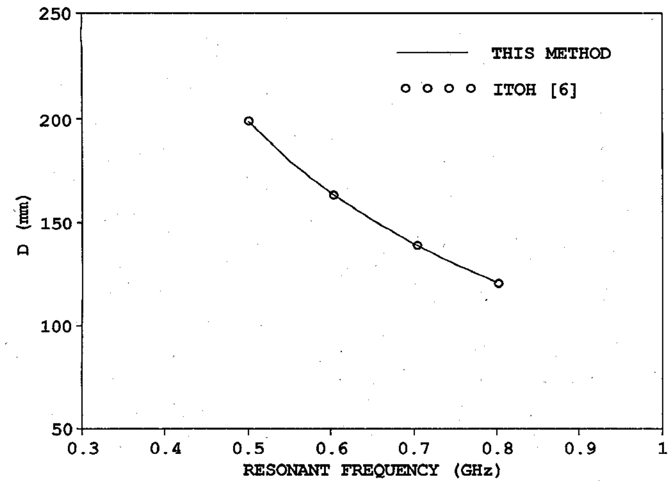

Fig. 2. Resonant frequency versus the resonator length with $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = 2.65$, $b = 155.0$ mm, $h_1 = 12.7$ mm, $h_2 = 88.9$ mm, and $w = 20.0$ mm.

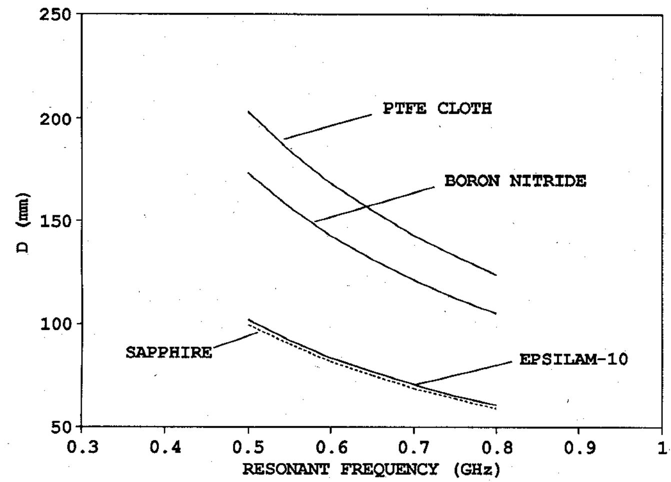

Fig. 3. Resonant frequency of the microstrip resonator printed on anisotropic substrates with $b = 155.0$ mm, $h_1 = 12.7$ mm, $h_2 = 88.9$ mm, and $w = 20.0$ mm.

Next, the data for microstrip resonators printed on anisotropic substrates are computed. For the biaxial substrate, namely PTFE cloth, the medium parameters are $\epsilon_{xx} = 2.45$, $\epsilon_{yy} = 2.89$, and $\epsilon_{zz} = 2.95$. For the uniaxial cases, they are specified as $\epsilon_{xx} = 3.4$ and $\epsilon_{yy} = \epsilon_{zz} = 5.12$, $\epsilon_{xx} = 10.3$ and $\epsilon_{yy} = \epsilon_{zz} = 13.0$, and $\epsilon_{xx} = 11.6$ and $\epsilon_{yy} = \epsilon_{zz} = 9.4$, written respectively for boron nitride, epsilam-10, and sapphire. Fig. 3 shows how the resonant frequency responds when various materials are used. Note that the physical parameters used here are the same as for those previously specified for the resonator printed on the isotropic substrate. The results show that by changing the medium from PTFE cloth to boron nitride, the resonant frequency reduces considerably, and this is in contrast to what happens when the medium changes from epsilam-10 to sapphire. In the second case, only a small variation in resonant frequency is observed. This behavior can be confirmed by using transmission line theory, where the resonant frequencies of the resonators are approximated by $(\lambda_o/2)/\sqrt{\epsilon_{\text{eff}}}$. Since the effective dielectric constant of an infinitely long microstrip line on epsilam-10 is very close to that computed for the sapphire substrate, this implies that the resonant frequencies of the resonators for these two media are just about the same.

IV. Conclusion

An analysis based on the spectral domain method applied to study shielded microstrip resonators printed on anisotropic materials was presented. The anisotropic layer is generally specified by its second rank permittivity tensor. The Green's function for the structure is obtained through a fourth order D. E. formulation. Galerkin's method testing procedure in the Fourier domain is applied to form the characteristic equation from which the resonant frequency of the resonator is numerically obtained. Data on the resonant frequency of resonators printed on both uniaxial and biaxial substrates were also generated.

References

[1] M. V. Schneider, "Millimeter-wave integrated circuits," *IEEE G-MTT Int. Symp. Microwaves*, June 1973.
[2] S. Mao, S. Jones, and G. D. Vendelin, "Millimeter-wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 455–461, July 1968.
[3] J. Wolff and N. Knoppik, "Rectangular and circular microstrip disc capacitors and resonators," in *Proc. European Microwave Conf.*, Brussels, Belgium, Sept. 1973.
[4] T. Itoh and R. Mittra, "Analysis of a microstrip disk resonator," *Arch. Eleck. Ubertagung.*, vol. 27, pp. 456–458, Nov. 1973.
[5] ——, "A technique for computing dispersion characteristics of shielded microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 896–898, Oct. 1974.
[6] T. Itoh, "Analysis of microstrip resonator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 946–952, Nov. 1974.
[7] T. Q. Ho and B. Beker, "Spectral domain analysis of shielded microstrip lines on biaxially anisotropic substrates," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1017–1021, June 1991.

Analysis of a Coupled Slotline on a Double-Layered Substrate Containing a Magnetized Ferrite

Masahiro Geshiro and Tatsuo Itoh

Abstract—**Nonreciprocity in the propagation characteristics of the even and odd modes in magnetized-ferrite-loaded double-layered coupled slotlines is studied. The analysis is based on Galerkin's method applied in the Fourier transform domain. Numerical results are presented for various values of structural parameters. As a result, it is found that the waveguide structures studied have sufficient nonreciprocity in propagation constants for isolators and four port circulators.**

I. Introduction

Slotlines are well suited for their usage in nonreciprocal ferrite devices for microwave integrated circuits since the magnetic field has elliptical polarization [1]. Recently, several analytical methods for propagation characteristics in slotlines and finlines containing

Manuscript received May 14, 1991; revised October 30, 1991.
M. Geshiro is with the Department of Electronics, Ehime University, Bunkyo-3, Matsuyama, Ehime 790, Japan.
T. Itoh is with the Department of Electrical Engineering, The University of California at Los Angeles, 66-147A Engineering IV, Los Angeles, CA 90024-1594.
IEEE Log Number 9106046.

0018-9480/92$03.00

a magnetized ferrite have been studied with intention of applying such waveguide structures to nonreciprocal components. For example, the ferrite finline isolator has been investigated by the field expansion method based on the TE-mode approximation [2]. More rigorous hybrid mode analyses based on mode-matching techniques [3], [4] and spectral-domain approaches [5]–[9] have also been reported. However, most papers published to data are concerned with uncoupled single transmission lines.

From the point of view of device designing, coupled lines can be utilized extensively for directional couplers, filters, and a wide variety of other devices. This paper presents an analysis of ferrite-loaded double-layered coupled slotlines with a pair of perfectly conducting planes as an enclosure. The analysis is based on the spectral domain method [10], one of the rigorous analytical methods, which is well-established and has been successfully used in the analysis of a wide variety of planar transmission line structures. The detailed mathematical description will be omitted in the following because the formulation is almost the same as in the paper published earlier [7].

II. Waveguide Structure and Analytical Method

The cross-sectional view of a ferrite-loaded double-layered coupled slotline under consideration and the coordinate system used for the analysis are shown in Fig. 1. Between the conductors and the ferrite layer of thickness d, an additional dielectric layer of thickness h is introduced which is reported to improve the nonreciprocal characteristics in propagation constant [2], [7]. Two perfect conductors parallel to each other are located at $x = \pm b$. It is found that these two plates have a negligible effect on the propagation characteristics when b is sufficiently larger than $S + 2W$. The permittivities ϵ_1 and ϵ_4 may be replaced with ϵ_0 in vacuum if those regions are air. The direction of propagation coincides with the z axis. It is assumed that the ferrite layer is magnetized by an external dc magnetic field applied in the x direction. Then, the tensor permeability of ferrite is expressed as

$$\bar{\bar{\mu}} = \mu_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mu_r & -j\kappa \\ 0 & j\kappa & \mu_r \end{pmatrix} \tag{1}$$

with

$$\mu_r = 1 - \frac{4\pi Ms\gamma^2 H_0}{\omega^2 - (\gamma H_0)^2}$$

$$\kappa = \frac{4\pi Ms\gamma\omega}{\omega^2 - (\gamma H_0)^2} \tag{2}$$

where μ_0, ω, H_0, $4\pi Ms$, and γ are the free-space permeability, the operating angular frequency, the external dc magnetic field, the magnetization of the ferrite, and the gyromagnetic ratio, respectively.

The spectral-domain field components $\tilde{E}x$ and $\tilde{H}x$ in the ferrite layer must satisfy the following equations:

$$\frac{d^2}{dy^2}\tilde{E}x_2 - (\alpha_n^2 + \beta^2 - k_2^2\mu_e)\tilde{E}x_2 = -j\omega\mu_0\xi\alpha_n\tilde{H}x_2$$

$$\frac{d^2}{dy^2}\tilde{H}x_2 - \left(\frac{1}{\mu_r}\alpha_n^2 + \beta^2 - k_2^2\right)\tilde{H}x_2 = j\omega\epsilon_2\xi\alpha_n\tilde{E}x_2 \tag{3}$$

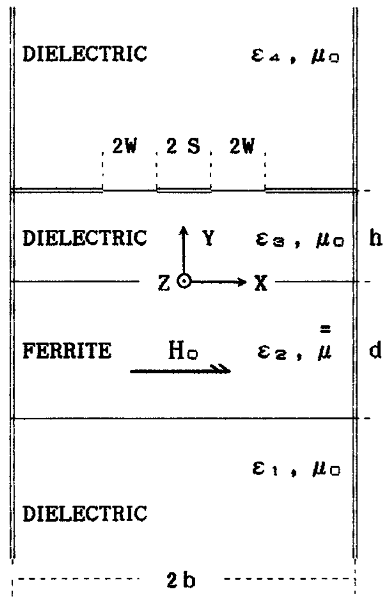

Fig. 1. Cross-sectional view of the ferrite-loaded double-layered coupled slotline.

where

$$k_2^2 = \omega^2\epsilon_2\mu_0 \tag{4}$$

$$\mu_e = \mu_r - \frac{\kappa^2}{\mu_r} \tag{5}$$

$$\xi = \frac{\kappa}{\mu_r}. \tag{6}$$

$\tilde{E}x$ and $\tilde{H}x$ are the Fourier transforms of Ex and Hx, which are defined as

$$\tilde{E}x_2(\alpha_n, y) = \int_{-b}^{b} Ex_2(x, y)\, e^{j\alpha_n x}\, dx$$

$$\tilde{H}x_2(\alpha_n, y) = \int_{-b}^{b} Hx_2(x, y)\, e^{j\alpha_n x}\, dx \tag{7}$$

where $\alpha_n = 2\pi n/2b$, $n = 0, 1, 2, \cdots$ for Ez odd modes and $\alpha_n = (2n - 1)\pi/2b$, $n = 1, 2, 3, \cdots$ for Ez even modes. For $\kappa = 0$ and $\mu_r = 1$, the wave equations (3) are reduced to those in the air region or the dielectric layer:

$$\frac{d^2}{dy^2}\begin{pmatrix}\tilde{E}x_i \\ \tilde{H}x_i\end{pmatrix} - (\alpha_n^2 + \beta^2 - k_i^2)\begin{pmatrix}\tilde{E}x_i \\ \tilde{H}x_i\end{pmatrix} = 0 \tag{8}$$

$$k_i^2 = \omega^2\epsilon_i\mu_0, \qquad i = 1, 3, 4. \tag{9}$$

The propagation factor $\exp[j(\omega t - \beta z)]$ has been omitted. Connecting the solutions in each region by applying the boundary conditions in the Fourier transform domain, we obtain

$$\begin{pmatrix} Yxx & Yxz \\ Yzx & Yzz \end{pmatrix}\begin{pmatrix}\tilde{E}x \\ \tilde{E}z\end{pmatrix} = \begin{pmatrix}\tilde{J}x \\ \tilde{J}z\end{pmatrix} \tag{10}$$

where $(\tilde{E}x, \tilde{E}z)$ and $(\tilde{J}x, \tilde{J}z)$ are the Fourier transforms of unknown electric field components in the slots and unknown current components on the conductors. The matrix elements, being known functions of α_n and β, are given in the Appendix of [7] in which $\coth\gamma_4(l_4 - h)$ and $\tanh\gamma_1(l_1 - d)$ should be replaced with unity.

In order to obtain the determinantal equation for the propagation constant, Galerkin's procedure is applied in the spectral domain.

$\tilde{E}x$ and $\tilde{E}z$ are expanded in terms of known basis functions as

$$\tilde{E}x = \sum_{i=1}^{M} c_i \tilde{\xi}_i(\alpha_n)$$

$$\tilde{E}z = \sum_{j=1}^{N} d_j \tilde{\eta}_j(\alpha_n) \qquad (11)$$

where c_i and d_j are unknown coefficients. The basis functions should be the Fourier transforms of appropriately chosen functions which are identically zero for $|X| < S$ and $S + 2W < |X| < b$. In actual computations, we choose

$$\xi_i(x) = \begin{cases} \dfrac{\cos\{(i-1)\pi(X-S-W)/2W + (i-1)\pi/2\}}{\sqrt{W^2-(X-S-W)^2}} \\ \quad + p\,\dfrac{\cos\{(i-1)\pi(X+S+W)/2W + (i-1)\pi/2\}}{\sqrt{W^2-(X+S+W)^2}}, & \begin{cases} i = 1, 2, 3, \cdots \\ S < |X| < S + 2W \end{cases} \\ 0, & \text{elsewhere} \end{cases} \qquad (12)$$

$$\eta_j(X) = \begin{cases} \dfrac{\sin\{j\pi(X-S-W)/W - j\pi\}}{\sqrt{W^2-(X-S-W)^2}} \\ \quad + p\,\dfrac{\sin\{j\pi(X+S+W)/W - j\pi\}}{\sqrt{W^2-(X+S+W)^2}}, & \begin{cases} j = 1, 2, 3, \cdots \\ S < |X| < S + 2W \end{cases} \\ 0, & \text{elsewhere} \end{cases} \qquad (13)$$

where $p = 1$ for the even modes and $p = -1$ for the odd modes. Applying Galerkin's procedure to (10) with (11), (12), and (13), we finally obtain a homogeneous equations system for the unknowns c_i and d_j, which is the determinantal equation for the propagation constants of the eigenmodes.

III. Numerical Results

Computational time critically depends on the number of basis functions used in the actual calculations. Table I shows the convergence of solutions on the number of basis functions for the odd mode propagating in the positive z direction at 40 GHz. The waveguide parameters have been chosen so that $\epsilon_1/\epsilon_0 = \epsilon_4/\epsilon_0 = 1$, $\epsilon_2/\epsilon_0 = \epsilon_3/\epsilon_0 = 12.5$, $4\pi Ms = 5000$ G, $H_0 = 500$ Oe, $d = h = 0.25$ mm, $2W = 2S = 0.5$ mm, and $2b = 10.0$ mm. The solutions are found to converge with a small number of basis functions. The table shows that the first two terms for Ex and one term for Ez in (11) are sufficient to obtain accurate solutions to three significant digits. It is found in the numerical computation that one thousand spectral terms of the discrete Fourier transforms are enough to maintain the above accuracy.

TABLE I
Convergence of Solutions at 40 GHz on the Number of the Basis Functions Where $\epsilon_1/\epsilon_0 = \epsilon_4/\epsilon_0 = 1$, $\epsilon_2/\epsilon_0 = \epsilon_3/\epsilon_0 = 12.5$, $4\pi Ms =$ 5000 G, $H_0 = 500$ Oe, $d = h = 0.25$ mm, $2W = 2S = 0.5$ mm, and $2b = 10.0$ mm.

N/M	1	2	3	4	5
1	2.89558	2.88397	2.88341	2.88304	2.88300
2	2.89564	2.88401	2.88344	2.88307	2.88302
3	2.89566	2.88402	2.88345	2.88309	2.88304

Fig. 2 shows the dispersion characteristics of the propagation constants of both even and odd modes with the slot width as a parameter. The solid lines represent the characteristics of the forward propagating waves, whereas the dashed lines are those of the backward propagating waves. The waveguide parameters except for the slot width have the same values as in the previous example. In this case, nonreciprocity between counter-propagating waves for the even mode is larger than that for the odd mode. This is a very interesting feature which implies a high potential of the waveguide structure on practical nonreciprocal devices.

Propagation constants of the four modes at 40 GHz are shown in Fig. 3 as a function of width $2S$ of the center strip, where $2W =$ 0.5 mm in (a) and $2W = 1.0$ mm in (b). The other parameters have the same values as in the example of Table I. From $\Delta\beta$, the difference in propagation constants of the even and odd modes, we can estimate the coupling coefficient between the two lines or the distance necessary for the complete transfer of power from one line to another line in a coupled transmission line. It is found from the figure that $\Delta\beta_+$ of the forward propagating waves is larger than $\Delta\beta_-$ of the backward propagating waves and that the ratio of $\Delta\beta_+$ to $\Delta\beta_-$ becomes fairly large for small values of $2S$.

The $\Delta\beta$'s at 40 GHz are shown in Fig. 4(a) as a function of thickness h of the dielectric substrate and 4(b) as a function of thickness of the ferrite layer. The other waveguide parameters have the same values as in the first example. It is worth noting that the ratio of $\Delta\beta_+$ and $\Delta\beta_-$ can be controlled over a wide range by changing either h or d.

If the ratio of $\Delta\beta_+$ and $\Delta\beta_-$ takes proper values, certain nonreciprocal devices are possible with the present waveguide structure of an appropriate length L, which can be regarded as a four port circuit. To be more specific, when $\Delta\beta L$ is π times an even integer for a forward wave and π times an odd integer for a backward one, the straight through state can be achieved by the former and the crossover state by the latter. This is the function just requisite to circulators and isolators. The above condition is satisfied, for example, around $d = 0.5$ mm in Fig. 4(b), though L should be almost ten times as long as the wavelength in the structure. It would seem, however, possible that the waveguide length necessary for the function can be made shorter by optimizing the structural parameters.

IV. Conclusions

Nonreciprocal propagation characteristics in double-layered coupled slotlines with a magnetized ferrite are analyzed by means of Galerkin's method in the spectral domain. Various numerical examples are presented for the propagation constants of the coun-

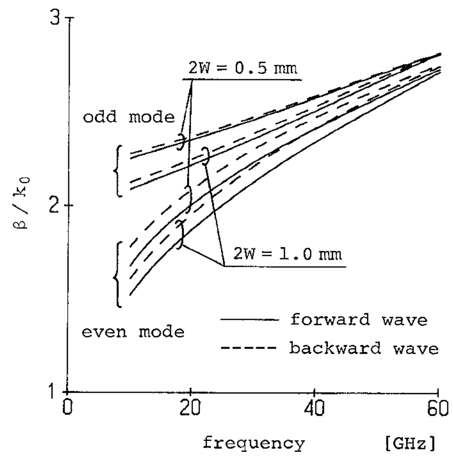

Fig. 2. Even and odd mode propagation constants versus frequency.

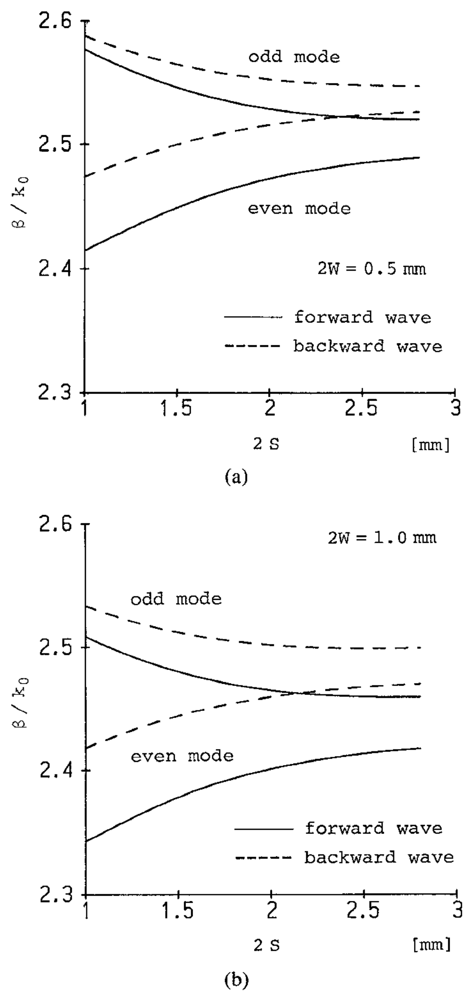

Fig. 3. Even and odd mode propagation constants versus width of the center strip, where $2W = 0.5$ mm in (a) and $2W = 1.0$ mm in (b).

terpropagating even and odd modes. In coupled transmission lines, $\Delta\beta$, the difference in propagation constants of the even and odd modes plays an important role. It is found from the numerical examples that we can obtain sufficient nonreciprocity in $\Delta\beta$ for application of these waveguide structures to such nonreciprocal devices as isolators and circulators. It is also found that the nonreciprocity can be controlled over a wide range by changing the thickness of the dielectric substrate.

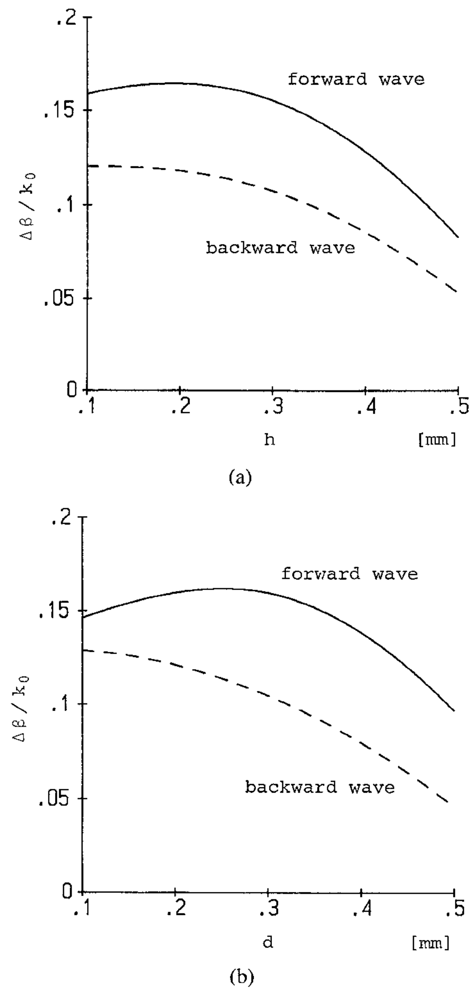

Fig. 4. Difference between even and odd mode propagation constants versus (a) thickness of the dielectric substrate and (b) thickness of the ferrite layer.

References

[1] S. B. Cohn, "Slot line on a dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 768–778, Oct. 1969.

[2] A. Beyer and K. Solback, "A new fin-line ferrite isolator for integrated millimeter-wave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 1344–1348, Dec. 1981.

[3] F. J. K. Lange, "Analysis of shielded strip- and slot-lines on a ferrite substrate transversely magnetized in the plane of the substrate," *Arch. Elek. Ubertragung*, vol. 36, pp. 95–100, Mar. 1982.

[4] G. Bock, "Dispersion characteristics of slot line on a ferrite substrate by a mode-matching technique," *Electron. Lett.*, vol. 18, pp. 536–537, Mar. 1982.

[5] J. Mazur and K. Grabowski, "Spectral domain analysis of multilayered transmission lines with anisotropic media," in *Proc. URSI Symp. on EMW*, Munich, 1980.

[6] Y. Hayashi and R. Mittra, "An analytical investigation of finlines with magnetized ferrite substrate," *IEEE Trans., Microwave Theory Tech.*, vol. MTT-31, pp. 495–498, June 1983.

[7] M. Geshiro and T. Itoh, "Analysis of double-layered finlines containing a magnetized ferrite," *IEEE Trans., Microwave Theory Tech.*, vol. MTT-35, pp. 1377–1381, Dec. 1987.

[8] M. Geshiro, Y. Takechi, and S. Sawa, "Propagation characteristics in double-layered coupled slotlines with a magnetized ferrite," in *Proc. Japan-China Joint Meeting on OFSET, EMT-90-121*, Oct. 1990.

[9] T. Kitazawa, "Analysis of shielded striplines and finlines with finite metallization thickness containing magnetized ferrites," *IEEE Trans., Microwave Theory Tech.*, vol. 39, pp. 70–74, Jan. 1991.

[10] T. Itoh and R. Mittra, "Spectral-domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Trans., Microwave Theory Tech.*, vol. MTT-21, pp. 496–499, July 1973.